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Modeling of Russian–Ukrainian war based on fuzzy cognitive map with genetic tuning

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Abstract

The Russian–Ukrainian conflict is considered as a dynamic system, whose variables are factors affecting the losses of the Russian army and the threat of the use of nuclear weapons. A fuzzy cognitive map (FCM) is used for modeling, that is, a directed graph whose vertices are model variables, and the weights of arcs are the degrees of positive and negative influences of variables on each other. The following factors influencing the losses of the Russian army and the threat of a nuclear strike were selected: resistance of the Ukrainian army, support of Ukraine with weapons, economic sanctions against Russia, opposition to the Russian government and its self-preservation instinct. The degrees of the influence of factors on each other and on the possibility of using nuclear weapons are evaluated by experts using fuzzy terms, which correspond to numeric values. To adjust the FCM, a genetic algorithm is used to select the degrees of influence of factors that minimize the discrepancy between the simulation results and expert estimations. The obtained FCM is used for scenario modeling of the conflict according to the "what if" scheme and ranking of factors according to their degree of influence on the level of nuclear threat.

Keywords

Russian–Ukrainian conflict, modeling, fuzzy cognitive map, genetic algorithm, scenario modeling, nuclear threat, ranking of influencing factors, pair effects

1. Introduction

On the night of 24 February 2022, the Kremlin announced a ''special military operation in Donbass,'' unleashing a full-fledged and aggressive war with Ukraine. Much of the world rallied in opposition to the Kremlin's aggressive plans, showing unprecedented solidarity with Ukrainians. An attempt to quickly capture the neighboring country failed. In response to Russian aggression, the West began supplying weapons to Ukraine and announced economic sanctions against Russia. This lead to an increase in losses of the Russian army and the emergence of anti-war sentiments in the Russian society. Russia possesses nuclear weapons, the use of which, according to leading political experts, is possible if Russia fails to achieve the goals of aggression with conventional weapons. The factors constraining the nuclear threat are the opposition to the Russian government caused by economic sanctions and its self-preservation instinct, that is, the fear of a retaliatory nuclear strike from the West.

Military and combat operations are historically the first object of modeling in science, which is engaged in operations research[1,2] and game theory.[3] Mathematical methods of quantitative substantiation of solutions in this area can be found in the reviews.[4,5] Judging by the applied developments,[6] at the strategic level of decision-making, the

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differential equations of Lanchester[7] are most widely used, they simulate the dynamics of changes in losses of each of the belligerents, depending on the parameters of the combat effectiveness of weapons and the rate of entry of reserves. Lanchester models have the following disadvantages:

- The parameters of combat effectiveness included in the equations are based on statistical data that are difficult to obtain, and the reliability of which may be questionable. Similar problems arise in the statistical theory of system reliability,[8] where the absence of failure rates for new elements causes distrust of reliability calculations.[9]
- Factors (technical, political, economic, etc.) affecting the dynamics of losses are not included in F. Lanchester's equations. This does not allow us to vary such factors as controlled variables when modeling various scenarios of the development of military operations.
- The equations are not adapted to work with expert opinions, which are almost the only source of information for modeling under uncertainty.

An alternative to differential equations modeling the dynamics of the system is fuzzy cognitive maps (FCMs) which became widespread after the publication of few works based on it.[10,11] A primer on the use of FCM may be found in literature.[12–15] An FCM is a directed graph with weighted arcs. The vertices of the graph are the variables which are taken into account in the model, and the weights of the arcs are the degrees of the variables' influence on each other. The step-by-step dynamics of variable values is calculated using a recurrence relation resembling an ordinary Markov chain.[1] Unlike the Markov chain, which uses the probabilities of states and transitions between them, an FCM uses the levels of values of variables and the degrees of their influences, which are described by membership functions of fuzzy sets.[16] The use of fuzzy mathematics in the FCM provides the convenience of modeling the dynamics of systems with qualitative variables measured by experts. In addition, the *principle of incompatibility of high complexity with high accuracy* is respected.[16] It is interesting to note that models similar to the FCMs were considered in the book[17] long before the publications;[10,11] however, the book[17] is not mentioned in numerous works on FCMs.

The expediency of using FCMs for modeling military conflicts follows from the following analogies:

- Connection of F. Lanchester's equations[7] with the method of dynamics of averages,[1] which directly follows from Markov processes, is shown by Wentzel[1];
- According to Wentzel,[1] the method of dynamics of averages used to simulate combat operations is directly related to ''predator–prey'' models in population dynamics[18];
- The possibility of modeling population dynamics using the FCM is shown by Dickerson and Kosko.[19]

Thus, the use of the FCM is a natural alignment of the development of the theory of modeling military operations based on the equations of F. Lanchester.[7]

It should be noted that FCM is a relatively new tool for mathematical modeling. Therefore, there are still far fewer applications of FCM in the field of military modeling than the number of applications of classical methods of operations research.[4–6] The main ideas of using FCM for modeling military-political systems are contained in the fundamental work of V. Kosko.[10] These ideas were used for scenario modeling the crisis of the former Yugoslavian Republic of Macedonia[20] and the political-economic problem of Cyprus.[21] Perusich and McNeese[22] propose the use of FCM as a technique for supporting the decision-making process in effect-based military planning with an illustrative example of an operation using NATO to stabilize a country. The FCM-based technique for mediating the information made available to decision makers in Airborne Warning and Control System crew managing air assets in conflict situation are proposed in Perusich and McNeese.[23] Jones et al.[24] describe a fuzzy cognitive model to emulate the decision support system for army infantry platoon leaders. The application of FCM for the modeling of complex structure of public support for insurgency and terrorism is described by Osoba and Kosko.[25]

In most applications of FCM in military modeling, expert assessments of the forces of influence of model variables (concepts) on each other are used. This does not guarantee that the predictions obtained using FCM will be close to the results of observations. Rotshtein and Katelnikov[26] proposed a two-stage approach for obtaining the weights of the arcs of the FCM graph: in the first stage, the admissible intervals of the weights of the arcs are determined; in the second stage, the weights of the arcs are tuned according to the results of observations using the least squares method.

The purpose of this article is to show the use of FCM for modeling military–political systems on the example with the Russian–Ukrainian conflict. Taking into account the potential capabilities of one of the belligerents, the main emphasis in modeling is on predicting the level of nuclear threat and ranking the factors influencing it. The usefulness and necessity of ranking factors (causes) affecting some objective function (consequence) is noted by Nechiporenko[27]—one of the first monographs on the structural analysis of systems under uncertainty.

Table 1. Assessment of the degree of influence.

Thermometer scale	Linguistic estimates	Numerical values
	Positive maximum	1
	Positive above average	0.75
	Positive average	0.5
	Positive below average	0.25
	Absent	0
	Negative below average	–0.25
	Negative average	–0.5
	Negative above average	–0.75
	Negative maximum	–1

When writing this article, the experience of designing and tuning FCM for the empirical modeling of reliability, safety, and risk assessment in human–machine systems was used.[28–30] This article consists of two parts: Section 1 contains the basic concepts and algorithms necessary for the application of the FCM; simulation results are provided in Section 2.

2. Modeling methodology

2.1. General information

According to Kosko,[10] the FCM is an oriented graph whose arcs are weighted by fuzzy terms.[16] The vertices of the graph, called *concepts*, correspond to the variables that are taken into account in the model, and the weights of the arcs reflect the *degrees of influence* of changes in *cause-variables* on changes in *effect-variables*. The term ''cognitive'' emphasizes that the initial data for modeling are subjective expert opinions about the degrees of influences, expressed in words such as ''rising'' or ''falling.'' The term ''fuzzy'' suggests that the FCM uses different levels of ''increase'' and ''decrease,'' which are given by numbers from the intervals [0,1] and $[-1, 0]$, which corresponds to the terms *weak*, *medium*, *strong*, and so on from the theory of fuzzy sets.[16]

2.2. Concepts

Let $C = \{C_1, C_2, ..., C_n\}$ is a known set of concepts, that is, variables used in the model. Each concept $C_i \in C$ will be considered a linguistic variable,[16] which is set on the universal set $[\underline{x_i}, \overline{x_i}]$ and is evaluated by the membership function $\mu_T(x_i)$ of the value $x_i \in [\underline{x_i}, \overline{x_i}]$ to the fuzzy term T, where $\underline{x_i}(\overline{x_i})$ is the lower (upper) boundary of the interval of acceptable values of the concept C_i.

2.2.1. Remark. In the works on the FCM, for example,[12–14] the concept is estimated by a number from the interval [0,1], which reflects the degree of membership to some fuzzy term. In this work, the concept C_i is estimated by the number $x_i \in [\underline{x_i}, \overline{x_i}]$, which is determined by one of their ways: (a) by an expert directly or (b) by defuzzification of the expert evaluation of the concept in the form of a fuzzy term T.

2.3. Connections between concepts

The weight w_{ij} of the arc connecting the concepts C_i and C_j indicates the degree of the influence C_i on C_j. Let concepts C_i and C_j are characterized by variables x_i and x_j, and as a result of the experiment, it is possible to build a dependency $x_j = \phi(x_i)$. Then the weight w_{ij} is defined as a derivative $w_{ij} = dx_j/dx_i$, which can be one of three types:

$w_{ij} > 0$, if an increase (decrease) in the value x_i leads to an increase (decrease) in the value x_j (the positive influence of C_i on C_j);

$w_{ij} < 0$, if an increase (decrease) in the value x_i leads to an decrease (increase) in the value x_j (the negative influence C_i on C_j);

$w_{ij} = 0$, if the value x_j does not depend on the value x_i (the absence of the influence of C_i on C_j).

The degree of influence of (w_{ij}) will be assessed expertly using linguistic terms and a thermometer scale (Table 1). If the opinions of several experts are taken into account, then the value w_{ij} is estimated as a weighted average of the estimates of each of the experts.

The practical possibility of expert evaluation w_{ij} is directly related to the remarkable ability of the human eye to detect linear dependencies in extrapolation problems.

2.4. Recurrence relation

The following concepts are used to describe the oscillatory process in the FCM:

- $(n \times n)$—is a *matrix of influences* of the concept C_i on concept C_j, in which the diagonal elements are 0, that is,

$$\mathbf{W}_0 = \begin{bmatrix} 0 & w_{12} & ... & w_{1n} \\ w_{21} & 0 & ... & w_{2n} \\ . & . & ... & . \\ w_{n1} & w_{n2} & ... & 0 \end{bmatrix} \quad (1)$$

- *The initial state* of the FCM, determined by the vector

$$\boldsymbol{X}^0 = \left[x_1^0,\ x_2^0, \ldots, x_n^0\right] \quad (2)$$

whose elements are equal to the values of the concepts at step $k = 0$.

- *The stationary state* of the FCM, determined by the vector

$$\boldsymbol{X}^l = \left[x_1^l, x_2^l, \ldots, x_n^l\right] \quad (3)$$

at this step l, when, as a result of the interaction between the concepts, the FCM enters a steady-state mode in which $|x_i^l - x_i^{l-1}| < \varepsilon$, where ε is a small positive number, $i = 1, 2, ..., n$.

2.4.1. Remark. The stationary state to which the oscillatory process converges in the FCM may correspond to one of the types of stability:[31] focus, orbit, or chaotic attractor.

To obtain a recurrence relation modeling, the dynamics of the FCM, we will use increments of the values of concepts in steps $k = 0, 1, 2, ...$ The value of the concept C_i in the step $(k + 1)$ depends on the values of the concepts C_j $(j = 1, 2, \ldots, n)$ in the previous step k. Let us denote this dependence as

$$x_i^{k+1} = \Psi\left(x_1^k, \ldots, x_j^k, \ldots x_n^k\right). \quad (4)$$

From equation (4) follows the relationship of increments (Δ) of the values of concepts at adjacent steps k and $(k + 1)$:

$$\Delta x_i^{k+1} = \frac{\partial x_i^{k+1}}{\partial x_1^k}\Delta x_1^k + \cdots + \frac{\partial x_i^{k+1}}{\partial x_j^k}\Delta x_j^k + \cdots + \frac{\partial x_i^{k+1}}{\partial x_n^k}\Delta x_n^k \quad (5)$$

The partial derivatives in equation (5) correspond to the degrees of the concepts' influences on each other $\partial x_i^{k+1}/\partial x_j^k = w_{ji}$. Therefore, the ratio equation (5) can be written as

$$\Delta x_i^{k+1} = \sum_{j=1}^{n} \Delta x_j^k w_{ji}, \quad i = 1,\ 2, \ldots, n, \quad (6)$$

where

$$\Delta x_i^{k+1} = x_i^{k+1} - x_i^k,\ \ \Delta x_i^k = x_i^k - x_i^{k-1}. \quad (7)$$

Taking into account equations (6) and (7), we obtain the equation of dynamics of step-by-step changes in the values of concepts:

$$x_i^{k+1} = x_i^k + \sum_{j=1}^{n}\left(x_j^k - x_j^{k-1}\right)w_{ji}. \quad (8)$$

The relation equation (8) can be represented in matrix form:

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^k \oplus \left(\boldsymbol{X}^k \ominus \boldsymbol{X}^{k-1}\right)\boldsymbol{W}_0, \quad (9)$$

where «$\oplus$» and «$\ominus$» are the operations of element-by-element addition and subtraction of vectors, which are performed according to the scheme:

$$[a, b] \oplus [c, d] = a + c,\quad b + d$$
$$[a,\ b] \ominus [c, d] = [a - c,\quad b - d].$$

In equation (9), it is assumed that for $k = 0$

$$\boldsymbol{X}^1 = \boldsymbol{X}^0 \oplus \boldsymbol{X}^0\boldsymbol{W}_{0.}$$

2.5. Prediction of the value of the output variable

Let us consider the input–output system, in which the C_n concept is an output variable, and the other concepts $C_1,\ C_2,\ \ldots,\ C_{n-1}$ are input variables that affect each other. Then the prediction of the value of the output variable is performed according to the following algorithm:

Step 1. Set the initial state of the FCM equation (2) by the vector

$$\boldsymbol{X}^0 = \left[x_1^0,\ \ x_2^0, \ldots, x_{n-1}^0,\ x_n^0 = 0\right],\ x_i \in \left[\underline{x_i}\ ,\ \overline{x_i}\right]. \quad (10)$$

Step 2. Using the recurrence relation equation (9), calculate the vector equation (3) of the values of concepts in a stationary state.
Step 3. In the resulting vector equation (3), fix the value of x_n^l and consider it as a forecast x_n of output, which corresponds to the specified input vector equation (10).

2.5.1. Remark. This algorithm is focused on estimating the mean value of the output variable level. To estimate the variation of output level, it is necessary to take into account the intervals of possible values of input variables.

The ranks of input concepts characterize their importance in terms of influencing the output concept. The methodology for ranking FCM concepts is proposed by Rotshtein et al.[29] The importance index of the input

Table 2. Observations "inputs—output."

No.	Inputs				Output
	C_1	C_2	…	C_{n-1}	C_n
1	x_{11}	x_{21}	…	$x_{n-1,1}$	x_{n1}
⋮	⋮	⋮	…	⋮	⋮
p	x_{1p}	x_{2p}	…	$x_{n-1,p}$	x_{np}
⋮	⋮	⋮	…	⋮	⋮
N	x_{1N}	x_{2N}	…	$x_{n-1,N}$	x_{nN}

concept C_j corresponds to the value $x_n^l = \hat{x}_n$ obtained for the vector equation (10), in which the value of the concept C_j is set at the upper level, and the remaining input concepts are set at the lower levels. Similarly, the indices of the joint influence of several input concepts are calculated.

2.6. Tuning of FCM

The task of tuning the FCM is similar to finding unknown parameters of an ordinary regression equation by the least squares method.[26] In the case of FCM, unknown parameters are the degrees of influences equation (1), which are set by experts and do not guarantee the coincidence of forecasts with experimental data. The essence of tuning is to improve expert estimates of the degrees of influences equation (1) based on observations of output values.

We will assume that as a result of observations, it is possible to collect the data presented in Table 2, where x_{ip} is the value of the concept C_i in the observation p, i = 1,2,..., n; p = 1,2,..., N, N is the number of observations. It is assumed that the values of x_{ip} in the Table 2 are obtained by the method of expert assessments. The parameters of the FCM, which are adjusted based on the results of observations, are the weights of the arcs $w_{ij} \in [\underline{w}_{ij}, \bar{w}_{ij}]$ where $\underline{w}_{ij} \in (\overline{w}_{ij})$ is the lower (upper) boundary of the interval of acceptable values w_{ij}.

Denote $\hat{x_n} = F(\boldsymbol{X}_0, \boldsymbol{W}_0)$ is the input–output dependency model, which corresponds to the prediction algorithm described in Section 2.5. Using this model and Table 2, we will find deviations

$$\varepsilon_p = x_{np} - \hat{x_{np}}, \ \ p = 1, \ 2, \ \ldots, N, \qquad (11)$$

where x_{np} is the value of the output in the pth observation, $\hat{x_{np}}$ is the forecast of the output at the values of the inputs from the pth observation, that is,

$$\hat{x_{np}} = F(x_{1p}, \ \ x_{2p}, \ldots, \ x_{n-1,p}, \ \ x_{n,p} = 0, \ \ \boldsymbol{W}_0).$$

Following the least squares method adopted in regression analysis, we formulate the task of tuning of the FCM based on observations as follows: to find a matrix of influence degrees $\mathbf{W}_0 = [w_{ij}, \ i = 1, \ 2, ..., \ n, \ j = 1, \ 2, ..., \ n]$, whose elements satisfy the constraints $w_{ij} \in [\underline{w}_{ij}, \bar{w}_{ij}]$ and minimize the sum of squared deviations equation (11), that is,

$$S(\boldsymbol{W}_0) = \sum_{p=1}^{N} \left(x_{np} - F\left(\boldsymbol{X}_p, \boldsymbol{W}_0\right)\right)^2 \underset{\boldsymbol{W}_0}{\rightarrow} \min, \qquad (12)$$

where $\boldsymbol{X}_p = [x_{1p}, x_{2p}, \ldots, x_{n-1p}, \ x_{np} = 0], \ p = 1, 2, \ldots, N.$

The subsection 3.2 demonstrates the methodology for selecting the intervals of permissible values of arc weights and the genetic algorithm for solving the problem equation (12) on the example of FCM of the Russian–Ukrainian war.

3. Simulation results

3.1. The FCM of the military conflict

We assume that Ukraine is waging a just war of liberation, which should end with Ukraine's victory. Therefore, when choosing a set of concepts, it is necessary to take into account the factors associated with different countries, which accelerate or inhibit the victory of Ukraine (Table 3). The concepts used in the FCM of the Russian–Ukrainian conflict are presented in Table 4 indicating the intervals of possible values of variables. The list of concepts is selected expertly and reflects the authors' point of view on the modeling object. The construction of a detailed hierarchy of factors affecting the level of military confrontation between Russia and Ukraine is an independent scientific task, which is not considered in this work. Table 4 shows that the evaluation interval of concept C_7 differs from the evaluation intervals of concepts $C_1, C_2, \ldots, C_6$. This is due to the following considerations: We believe that the presence of nuclear weapons determines a continuous, that is, a constant level of risk of its use (0), which can be maximally reduced to (–25) or maximally increased to $(+25)$. Qualitative factors that must be taken into account during expert assessment of the level of each of the concepts, as well as the strength of their influence, are summarized in the Table 5.

The FCM graph is shown in Figure 1. The weights of the arcs of the graph reflecting the degrees of the concepts' influences were selected expertly taking into account Table 1. The matrix equation (1) of the degrees of influences has the form:

$$\boldsymbol{W}_0 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\ 0 & 0 & 0.4 & 0.5 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.45 & 0 & 0.75 \\ 0 & 0 & 0.45 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 0.55 & 0 & 0 \\ 0 & -0.5 & 0 & -0.25 & 0 & 0.75 & 0 \end{array} \qquad (13)$$

Table 3. Factors affecting the victory of Ukraine.

	Factors affecting the victory of Ukraine (end of war)	
	Positive impact	Negative impact
Ukraine	• Victories of the Armed Forces of Ukraine • The size of the army • The level of training of military personnel • Motivation of the army • Mastering new military technologies • Capture of trophy equipment • Support of the population	• Defeats of the Armed Forces of Ukraine • Infrastructure failure • Economic losses • Civilian casualties
Russia	• Defeats of the Russian army • Desertion, surrender • The destruction of decision-making centers and logistics centers in the occupied territories and in the Russian Federation • Fustration of the population	• Victories of the Russian army • The size of the Russian army • Level of training of military personnel of the Russian Federation • Motivation of the Russian army • Availability of weapons (offensive and defensive) • Support of the elite of the Russian Federation
Belarus		• Supply of weapons • Free access to air space • Political support
USA	• Supply of weapons (offensive and defensive) • Sanctions against the Russian Federation • Financial support • Political support	• Opposition to support Ukraine
Europe	• Supply of weapons (offensive and defensive) • Sanctions against the Russian Federation • Financial support • Political support • Support for refugees	• Opposition to support Ukraine • Dependence on Russian resources
China		Economic support
Iran		Supply of drones to Russia
Turkey	Supply of drones to Ukraine	

Table 4. Concepts in the conflict model

Concept	Content	Interval for estimation
1	Resistance of the Ukrainian army	[0, 50]
2	Support of the Ukrainian army with weapons	[0, 50]
3	Losses of the Russian army	[0, 50]
4	Economic sanctions against Russia	[0, 50]
5	Opposition to the Russian government	[0, 50]
6	The instinct of self-preservation of the Russian government	[0, 50]
7	The threat of a nuclear strike	[–25, 25]

For expert evaluation of variables corresponding to the concepts from Table 2, we will use the scales presented in Tables 6 and 7.

3.1.1. Remark. The numerical values, which are located opposite the linguistic estimates in Tables 1, 2, and 5, correspond to the maxima of the membership functions of fuzzy terms.[16]

The matrix equation (13) together with the recurrence relation equation (9) allows us to observe a step-by-step change in the values of concepts from Table 4 for a given initial vector equation (2).

In the example in Figure 2, at the initial state of the FCM

$$\boldsymbol{X}_0 = [\,40 \quad 25 \quad 15 \quad 20 \quad 10 \quad 5 \quad 0\,]$$

we obtain a stationary state, which is determined by the vector.

$$\boldsymbol{X}_l = [\,52 \quad 20 \quad 55 \quad 43 \quad 63 \quad 12 \quad 10\,].$$

In this vector, the value $\hat{x}_7 = 10$ can be interpreted as "some increase in the risk of a nuclear threat."

3.1.2. Remark. The use of the recurrent relation equation (9) can lead to the fact that the accumulated values of concepts in a stationary state go beyond the intervals of acceptable values. Since this does not affect the qualitative nature of the conclusions, the normalization procedure described by Rotshtein and Katelnikov[26] is not used in this article.

Table 5. Factors taken into account in the expert assessment of the level of concepts.

Concept	Factors
C_1	• Motivation and fighting spirit of the Ukrainian army • Military activity • Liberation of the occupied territories
C_2	• Timely supply of weapons: tanks, airplanes, air defense equipment, artillery, etc. • Assistance in personnel training
C_3	• Failure to capture Kyiv • Excessive number of killed soldiers • Losses of military equipment
C_4	• Oil and gas export ban • Sanctions in the field of electronics • Individual sanctions against oligarchs • Withdrawal of Western companies from the Russian market • The collapse of the Russian economy
C_5	• Anti-war sentiments in society • Refusal to participate in mobilization • Decrease in motivation in the Russian army • Decrease in Putin's rating
C_6	• Fear of the consequences of a nuclear strike • Reluctance to follow commanders' orders • US threats to eliminate Putin in exchange for a nuclear attack
C_7	• Putin's statement on Russian television "Why do we need such a world if there is no Russia?" • Threats on Russian television "to turn the whole world into radioactive ash" • The desire of the "hawks" in Russian politics to take revenge with a nuclear attack for failures at the front

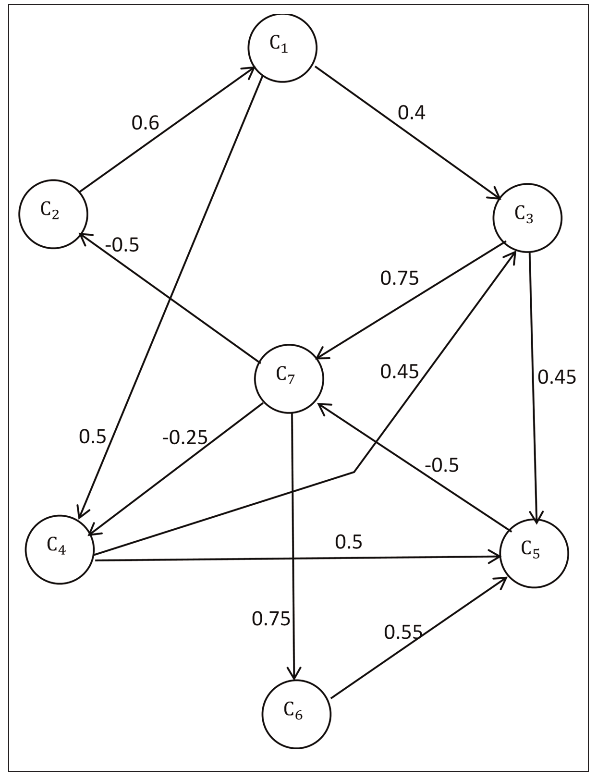


Figure 1. FCM of the military conflict.

Table 6. Assessment of the level of concepts C_1, C_2,..., C_6.

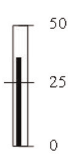
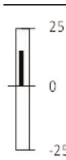
Scale	Linguistic evaluations	Numbers
	High Upper average Average Under average Low	50 37.5 25 12.5 0

Table 7. Assessment of the level of the concept C_7.

Scale	Linguistic evaluations	Numbers
	Maximum Higher Stable Lower Minimum	25 12.5 0 –12.5 –25

3.2. Tuning of FCM

3.2.1. Intervals of arc weights. The parameters of the FCM, which are tuned based on the results of observations, are the weights of arcs $w_{ij} \in [\underline{w}_{ij}, \bar{w}_{ij}]$, where $\underline{w}_{ij}$ $(\bar{w}_{ij})$ is the lower (upper) boundary of the interval of acceptable values w_{ij}. When choosing intervals $[\underline{w}_{ij}, \bar{w}_{ij}]$ we will proceed from the following assumptions:

— the type of influence $w_{ij} > 0$, $w_{ij} < 0$ or $w_{ij} = 0$ is determined by experts and does not change when tuning the FCM;

— the degree of influence w_{ij} is estimated by an expert with an accuracy of one linguistic term (see Table 1), that is, $\pm\ 0.2$;

— the degrees of positive $(w_{ij} > 0)$ and negative $(w_{ij} < 0)$ influences vary in the intervals [0.05, 0.95] and [–0.95, –0.05], respectively.

Taking into account these assumptions, the intervals of the weights of the FCM arcs are selected according to the scheme (see Table 8):

$$0.3 \in [0.1, 0.5],\ -0.3 \in [-0.5,\ -0.1],$$
$$0.1 \in [0.05, 0.3],\ -0.1 \in [-0.3,\ -0.05],$$
$$0.8 \in [0.6, 0.95],\ -0.8 \in [-0.95,\ -0.6].$$

```
begin
    t: = 0; to set the initial value of P(t);
    to evaluate P(t) using the fitness function;
    while (until the completion conditions are met) do
        to perform crossover operation on selected
        parents P(t), in order to get C(t);
        to perform mutation operation on offspring C(t);
        to evaluate C(t) using the fitness function;
        to choose P(t + 1) from P(t) and C(t);
        t: = t + 1;
     end
end.
```

Table 8. Intervals of changing the weights of arcs.

Weight	$\underline{w}_{ij}$	$\bar{w}_{ij}$
w_{13}	0.2	0.6
w_{14}	0.3	0.7
w_{21}	0.4	0.8
w_{35}	0.25	0.65
w_{37}	0.55	0.95
w_{43}	0.25	0.65
w_{45}	0.3	0.7
w_{57}	–0.7	–0.3
w_{65}	0.35	0.75
w_{72}	–0.7	–0.3
w_{74}	–0.45	–0.05
w_{76}	0.55	0.95

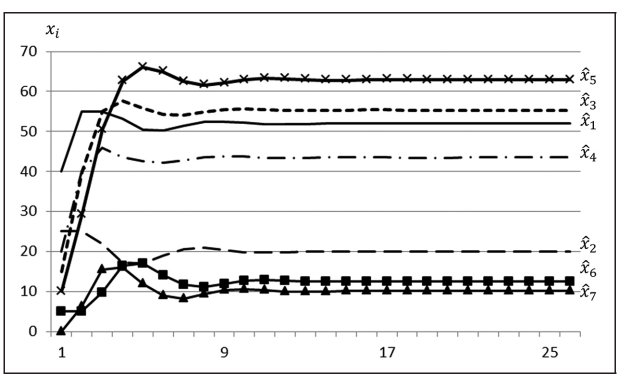


Figure 2. An example of the dynamics of changing the values of concepts for the initial vector $X^0 = [40,\ 25,\ 15,\ 20,\ 10,\ 5,\ 0]$.

3.2.2. The genetic algorithm. To solve the nonlinear optimization problem equation (12), a genetic algorithm is proposed based on the following concepts and operations:[30] *chromosome* is a coded solution; *population* is an initial set of solutions; *fitness function* is a criterion for selecting parents; *crossover* is the operation of generating chromosomes-offspring from the parent chromosomes; *mutation* is a random change in the elements of the chromosome.

If $P(t)$ are the parent chromosomes, and $C(t)$ are the offspring chromosomes at the tth iteration, then the general structure of the genetic algorithm has the form

A chromosome is defined as a vector of nonzero matrix elements $\mathbf{W}_0 = [w_{ij}]$, $w_{ij} = R[\underline{w}_{ij}, \bar{w}_{ij}]$, where $R[\underline{x}, \bar{x}]$ is the operation of finding a random number randomly distributed over an interval $[\underline{x}, \bar{x}]$. For example, for the FCM in Figure 1, the generator of the initial population of chromosomes is a vector $[w_{13}, w_{14}, w_{21}, w_{35}, w_{37}, w_{43}, w_{45}, w_{57}, w_{65}, w_{72}, w_{74}, w_{76}]$, where $w_{13} = R[0.2, 0.6]$, $w_{14} = R[0.3, 0.7]$, $w_{21} = R[0.4, 0.8]$, $w_{35} = R[0.25, 0.65]$, $w_{37} = R[0.55, 0.95]$, $w_{43} = R[0.25, 0.65]$, $w_{45} = R[0.3, 0.7]$, $w_{57} = R[-0.7, -0.3]$, $w_{65} = R[0.35, 0.75]$, $w_{72} = R[-0.7, -0.3]$, $w_{74} = R[-0.45, -0.05]$, and $w_{76} = R[0.55, 0.95]$. Hence the examples of chromosomes:

$$[0.40, 0.36, 0.54, 0.44, 0.64, 0.51, 0.64, -0.44, 0.71, -0.38, -0.19, 0.67] \tag{14}$$

$$[0.43, 0.69, 0.66, 0.41, 0.85, 0.26, 0.65, -0.63, 0.64, -0.40, -0.31, 0.65]. \tag{15}$$

The crossing of a pair of parent chromosomes generates an offspring chromosome. The crossing operation is performed by a random exchange of genes (elements) of the parent chromosomes. To do this, a random number is assigned to each gene of the offspring chromosome $\xi_1 = R[0, 1]$. If $\xi_1 \leq 0.5$, then this gene is taken from the first parent, otherwise the gene is taken from the second parent. Let the parent chromosomes be given by strings equations (14) and (15), and random numbers ξ_1 correspond to the vector [0.23, 0.54, 0.80, 0.14, 0.71, 0.58, 0.78, 0.06, 0.04, 0.06, 0.55, 0.30]. Then, as a result of crossing equations (14) and (15), we get the offspring chromosome

$$[0.40, 0.69, 0.66, 0.44, 0.85, 0.26, 0.65, -0.44, 0.71, -0.38, -0.31, 0.67]. \tag{16}$$

Each gene in equation (16) can undergo mutation. To do this, a random number $\xi_2 = R[0, 1]$ is assigned to each gene and the mutation coefficient q is set (in this case, $q = 0.1$). If $\xi_2 \leq q$ then this gene is replaced by a random number from the range of acceptable values.

Let a vector of random numbers ξ_2 have the form

$$[0.05,\ 0.66,\ 0.71,\ 0.01,\ 0.18,\ 0.83,\ 0.59,\ 0.68,\ 0.13,\ 0.43,\ 0.43,\ 0.78].$$

Table 9. Training data for the FCM.

No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	1	1	1	1	50	50	–25
2	5	5	5	5	40	40	–20
3	10	10	10	10	10	30	–10
4	20	20	20	20	20	20	0
5	30	30	30	30	10	10	10
6	40	40	40	40	5	5	20
7	50	50	50	50	1	1	25

Table 10. Arc weights before and after tuning.

Weight	Before tuning	After tuning
w_{13}	0.4	0.2
w_{14}	0.5	0.65
w_{21}	0.6	0.4
w_{35}	0.45	0.25
w_{37}	0.75	0.55
w_{43}	0.45	0.25
w_{45}	0.5	0.3
w_{57}	–0.5	–0.35
w_{65}	0.55	0.75
w_{72}	–0.5	–0.7
w_{74}	–0.25	–0.15
w_{76}	0.75	0.65

Table 11. Expert assessments and simulation results.

No.	x_7	Before tuning		After tuning	
		$\hat{x}_7$	$\lvert x_7 - \hat{x}_7\rvert$	$\hat{x}_7$	$\lvert x_7 - \hat{x}_7\rvert$
1	–25	–30.00	5.00	–25.01	0.01
2	–20	–21.22	1.22	–17.86	2.14
3	–10	–3.87	6.13	–4.38	5.62
4	0	0.97	0.97	0.00	0
5	10	13.72	3.72	10.20	0.2
6	20	23.40	3.40	17.86	2.14
7	25	32.47	7.47	25.00	0

Therefore, only the first and fourth genes are mutated in chromosome equation (16), and after that the offspring chromosome takes the form. [0.28, 0.69, 0.66, 0.08, 0.85, 0.26, 0.65, -0.44, 0.71, -0.38, -0.31, 0.67], where 0.28 = R[0.2, 0.6], 0.37 = R[0.25, 0.65].

The fitness function is criterion equation (12) with a minus sign, that is, the better the chromosome meets the optimization criterion, the greater the fitness function.

The selection of parent chromosomes for the crossing operation is not accidental. Priority is given to the best solutions. The larger the fitness function, the greater the probability that a given chromosome will give offspring.[30] When the genetic algorithm is executed, the population size remains constant. Therefore, after crossing operations and mutations, it is necessary to remove chromosomes from the resulting population that have the worst value of the fitness function.

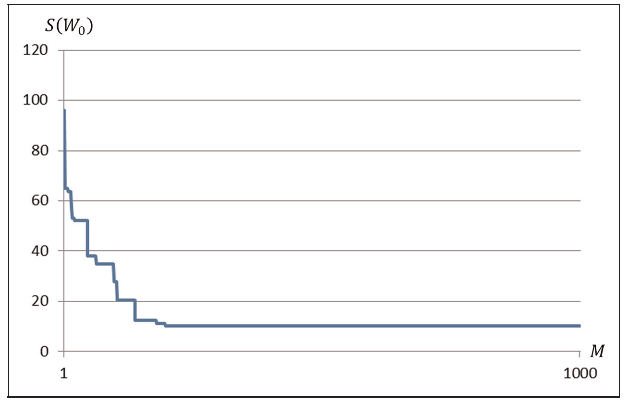


Figure 3. Dynamics of changes in the optimization criterion.

3.3. The results of tuning of the FCM

To tune the initial version of the FCM (Figure 1), the training data ''influencing factors – nuclear threat'' from the Table 9 were used.

These data were discussed expertly using linguistic assessments (Tables 6 and 7) and then transformed into digital values. As a result of solving the optimization problem equation (12), the weights of the arcs of the FCM graph after tuning are obtained, which are presented in Table 10. Comparison of expert values of the nuclear threat level (x_7) and simulation results ($\hat{x}_7$) before and after tuning of the FCM for seven scenarios is shown in Table 11, that is, mean absolute deviation (MAD) and the mean square error (MSE) are:

$$\mathrm{MAD} = \frac{1}{7}\sum_{p=1}^{7}\left|x_{7,p} - \widehat{x_{7,p}}\right| = \begin{cases} 3.99, & \text{before tuning} \\ 1.45, & \text{after tuning} \end{cases}$$

$$\mathrm{MSE} = \frac{1}{7}\sum_{p=1}^{7}\left(x_{7,p} - \widehat{x_{7,p}}\right)^2 = \begin{cases} 20.90, & \text{before tuning} \\ 5.83, & \text{after tuning} \end{cases}$$

Thus, the use of FCM with tuned arc weights can significantly improve the performance of MAD and MSE. To solve the optimization problem equation (12), a genetic algorithm was used when tuning of the FCM, which is described in detail by Rotshtein et al.[30] The dynamics of changes in the optimization criterion equation (12) with an increase in the number of iterations M is shown in Figure 3.

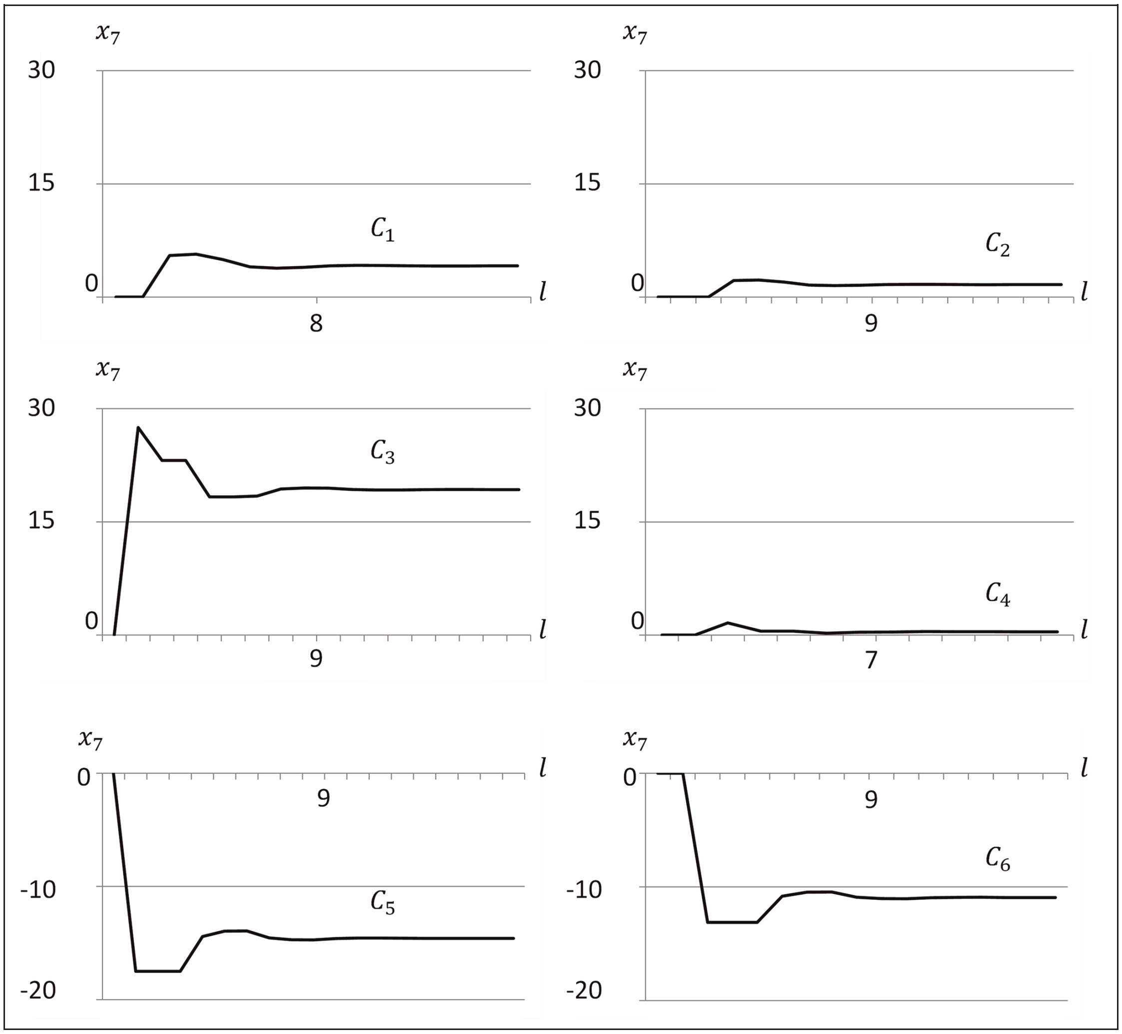


Figure 4. Step-by-step change in the value of the concept C_7 when calculating the indexes of the importance of concepts $C_1,\ C_2, \ldots, C_6$.

3.4. Ranking of factors affecting the nuclear threat

According to Rotshtein et al.,[29] the importance indices of concepts $C_1,\ C_2, \ldots, C_6$ in terms of their influence on the concept C_7 coincide with the values of x_7, which are calculated using the algorithm from Section 2.5 for the following initial vectors equation (10):

$\boldsymbol{X}_0 = [50,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]-$ for the influence of the concept C_1,

$\boldsymbol{X}_0 = [0,\ 50,\ 0,\ 0,\ 0,\ 0,\ 0]-$ for the influence of concept C_2,

...

$\boldsymbol{X}_0 = [0,\ 0,\ 0, 0,\ 0,\ 50,\ 0]-$ for the influence of the concept C_6.

For each of these vectors, Figure 4 shows a step-by-step change in the values of the C_7 concept before entering the stationary state. The obtained values of x_7 are presented in Figure 5 in the form of a diagram that shows the relative importance of factors $(C_1, C_2, \ldots, C_6)$, affecting the level of nuclear threat (C_7). Figure 5 shows that the greatest increase in the nuclear threat is associated with the losses of the Russian army (C_3), and the greatest decrease in this threat is caused by the opposition to the Russian government (C_5) and the instinct of its self-preservation (C_6).

To evaluate the paired effects of factors on the concept C_7, initial vectors of the following type were used:

$\boldsymbol{X}_0 = [50,\ 50,\ 0,\ 0,\ 0,\ 0,\ 0]-$ for joint influence of concepts C_1 and C_2,

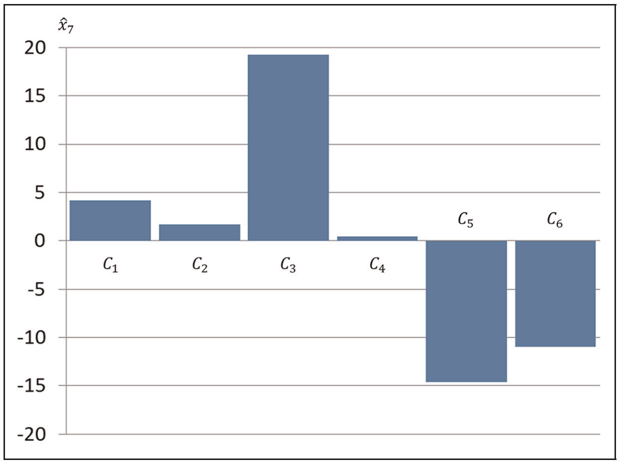


Figure 5. Diagram of influences on the C_7 concept.

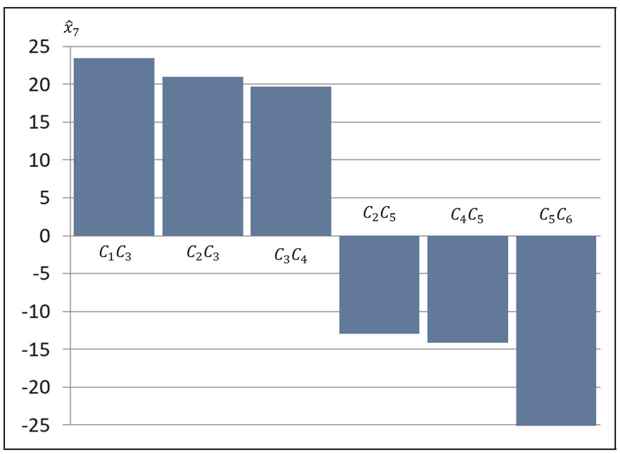


Figure 6. Diagram of paired influences on the C_7 concept.

$\boldsymbol{X}_0 = [50,\ 0,\ 50,\ 0,\ 0,\ 0,\ 0]$ – for joint influence of concepts C_1 and C_3,

...

$\boldsymbol{X}_0 = [0,\ 0,\ 0,\ 0,\ 50,\ 50,\ 0]$ – for joint influence of concepts C_5 and C_6,

The diagram of the most important paired influences is shown in Figure 6. It can be seen from this figure that the greatest increase in the nuclear threat is due to the joint influence of the Ukrainian resistance (C_1) and Russian losses (C_3), and the greatest decrease in this threat is caused by the Russian opposition (C_5) and the instinct of self-preservation of the Russian government (C_6).

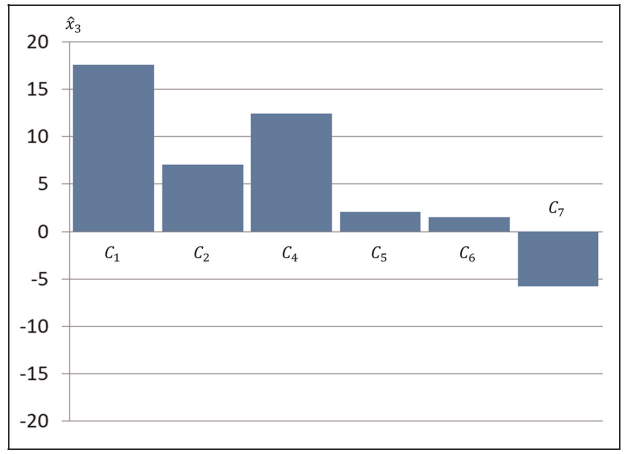


Figure 7. Diagram of influences on the C_3 concept.

3.5. Ranking of factors affecting the losses of the Russian army

Since the main factor affecting the level of nuclear threat is the losses of the Russian army (C_3), it is of interest to rank the factors affecting the C_3 concept. The importance indices of concepts $C_1,\ C_2,\ C_4, \ldots,\ C_7$ in terms of their influence on the concept C_3 coincide with the stationary values of $\hat{x}_3$, which are calculated with the following initial vectors:

$\boldsymbol{X}_0 = [50,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]$ – for the influence of the concept C_1,

$\boldsymbol{X}_0 = [0,\ 50,\ 0,\ 0,\ 0,\ 0,\ 0]$ – for the influence of the concept C_2,

$\boldsymbol{X}_0 = [0,\ 0,\ 0,\ 50,\ 0,\ 0,\ 0]$ – for the influence of the concept C_4,

...

$\boldsymbol{X}_0 = [0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 50]$ – for the influence of the concept C_7.

The obtained values of $\hat{x}_3$ are presented in Figure 7 as a diagram of the importance of the influence of factors $C_1,\ C_2,\ C_4, \ldots,\ C_7$ on the losses of the Russian army (C_3). Figure 7 shows that the greatest increase in Russian losses is associated with the Ukrainian resistance (C_1) and economic sanctions (C_4), and the greatest decrease in losses is caused by the nuclear threat (C_7). Similarly, a diagram of the paired effects of factors on the (C_3), concept is obtained, which is shown in Figure 8.

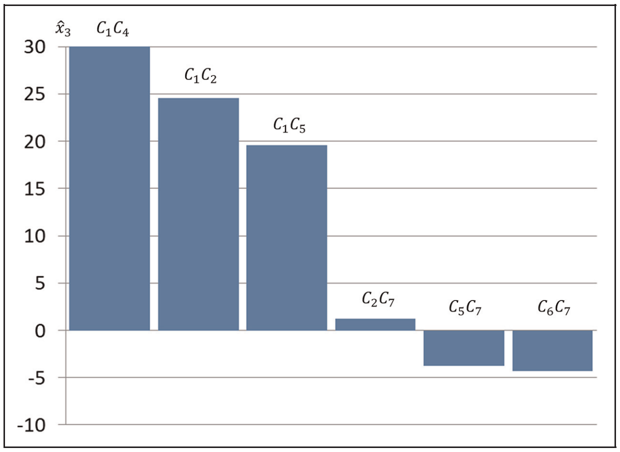


Figure 8. Diagram of paired influences on the C_3 concept.

4. Conclusion

The Russian–Ukrainian military conflict is an actual object of modeling by means of operations research and system analysis. The most important aspect of the simulation is the threat of the use of nuclear weapons, which is associated with the potential features of one of the parties to the conflict.

In this paper, it is shown that FCMs are analogous to differential equations traditionally used to model the dynamics of losses in military conflicts. The advantage of the FCM in comparison with differential equations is the possibility of using expert information to directly account for interrelated factors (military—technical, economic, political, etc.) that affect the dynamics of losses and the level of nuclear threat.

We have proposed a model of the Russian–Ukrainian conflict in the form of an FCM, taking into account factors related to Ukraine, Russia, and countries that support Ukraine. To tune the FCM, an expert training sample of seven scenarios reflecting different levels of increasing and decreasing nuclear threat was used. On the basis of the proposed FCM, indices of the importance of factors according to the degree of their influence on the increase or decrease in the level of nuclear threat and losses of the Russian army were obtained. The practical value of the modeling results lies in the fact that the obtained indices of the importance of factors (Figures 5–8) can be used in decision-making for reducing the level of nuclear threat.

A promising direction for further research is to expand the proposed FCM based on a detailed classification of factors affecting the development of the Russian–Ukrainian conflict.

The FCM, which displays typical elements, properties, and relations of all participants in the conflict, can be considered as some kind of scheme—an analogue of traditional engineering schemes: structural, functional, basic electrical, and so on.

Thus, it becomes possible to transfer well–developed engineering techniques to a relatively new field of military-political objects for the application of mathematics.

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