

CONFORMAL MAPPINGS OF RIEMANNIAN MANIFOLDS PRESERVING THE GENERALIZED EINSTEIN TENSOR

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Abstract. We study conformal mappings preserving the generalized Einstein tensor. We have derived corresponding partial differential equations and their integrability conditions. In addition to the generalized Einstein tensor we got other invariants of the mappings. Also we have proved that orientable compact manifolds equipped by positive definite metric, do not admit conformal mappings preserving the generalized Einstein tensor.

Keywords: Affine connection, Riemannian manifolds, conformal mappings, generalized Einstein tensor.

Mathematics subject classification: Primary 53B20; Secondary 53C21, 53B50

1 Introduction

Diffeomorphisms preserving certain geometric objects are being given much attention of many researchers in the differential geometry realm.

In particular, conformal mappings which preserved the Einstein tensor

$$E_{ij} = R_{ij} - \frac{Rg_{ij}}{n}$$

studied in [1]. Preserving the stress-energy tensor

$$S_{ij} = R_{ij} - \frac{Rg_{ij}}{2}$$

by conformal mappings was explored in [4], [2]. It's worth for noting that in many classical issues e. g. [7, p. 359], just the latter is referred to as the Einstein tensor. Let us refer to

$$\mathfrak{E}_{ij} \stackrel{\text{def}}{=} R_{ij} - \kappa Rg_{ij}. \quad (1)$$

as **the generalized Einstein tensor**. Here κ is a constant.

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