An Intuitionistic Fuzzy Approach to Analysis Financial Risk Tolerance with MATLAB in Business

Vinesh Kumar, Sandeep Kumar Gupta, Rohit Kaushik, Subhask Kumar Verma, and Olena Sakovska

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [1] in 1965. Later in 1983, Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. There are so many applications of intuitionistic fuzzy sets in different fields. In this paper, we consider the risk tolerance capacity in business [3]. Risk tolerance relates to the amount of market risk such as volatility, market ups and downs which can be tolerated by an investor. The financial service institutions aim to help the financial planner build a portfolio of investment that the investor will be comfortable with over a long period. Every investor needs to measure their risk tolerance (RT) before choosing their investment.

In this research paper, we present a model of an investor's risk tolerance capacity which depends on his/her current income (CI) and total net worth (TNW).

V. Kumar
Phonics Group of Institutions, Roorkee, India

S. K. Gupta (✉) · S. K. Verma
AMET University, Chennai, India
e-mail: skguptabhu@gmail.com

R. Kaushik
G. L. Bajaj Institute of Technology and Management, Greater Noida, India

S. K. Verma
School of Business Management, Noida International University, Greater Noida, India

O. Sakovska
Uman National University of Horticulture, Uman, Ukraine

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2 Preliminaries

In this section, we review some elementary concepts related to this paper.

Definition 2.1 [1] Let X be a non-empty set. A fuzzy set $\widetilde{A}$ drawn from X is defined as

$$\widetilde{A} = \left\{ \left\langle x, \mu_{\widetilde{A}}(x) \right\rangle : x \in X \right\}, \tag{2.1}$$

where $\mu_{\widetilde{A}}(x) : X \to [0, 1]$ is a membership function of the fuzzy set $\widetilde{A}$.

Definition 2.2 [2] An Intuitionistic fuzzy set $\widetilde{A}^i$ on X is given by

$$\widetilde{A}^i = \left\{ \left\langle x, \mu_{\widetilde{A}^i}(x), \nu_{\widetilde{A}^i}(x) \right\rangle : x \in X \right\}, \tag{2.2}$$

where $\mu_{\widetilde{A}^i}(x) : X \to [0, 1]$ and $\nu_{\widetilde{A}^i}(x) : X \to [0, 1]$ such that $0 \leqslant \mu_{\widetilde{A}^i}(x) + \nu_{\widetilde{A}^i}(x) \leqslant 1, \forall x \in X$.

The value of $\mu_{\widetilde{A}^i}(x)$ is a lover bond on the degree of membership of x derived from the evidence for x, and $\nu_{\widetilde{A}^i}(x)$ is a lower bond on the negation of x derived from the evidence x.

We call $\pi_{\widetilde{A}^i}(x) = 1 - \mu_{\widetilde{A}^i}(x) - \nu_{\widetilde{A}^i}(x), x \in X$ to be hesitation or the intuitionistic index of x in $\widetilde{A}^i$. This index indicates the lack of knowledge to fact whether the element belongs to the set or not.

For two intuitionistic fuzzy sets, $\widetilde{A}^i$ and $\widetilde{A}^i$ in X it hold that

- $\widetilde{A}^i \cap \widetilde{B}^i = \left\{ \left\langle x, \min\left(\mu_{\widetilde{A}^i}(x), \mu_{\widetilde{B}^i}(x)\right), \max\left(\nu_{\widetilde{A}^i}(x), \nu_{\widetilde{B}^i}(x)\right) \right\rangle : x \in X \right\}$
- $\widetilde{A}^i \cup \widetilde{B}^i = \left\{ \left\langle x, \max\left(\mu_{\widetilde{A}^i}(x), \mu_{\widetilde{B}^i}(x)\right), \min\left(\nu_{\widetilde{A}^i}(x), \nu_{\widetilde{B}^i}(x)\right) \right\rangle : x \in X \right\}$
- $\widetilde{A}^i \subset \widetilde{B}^i$ iff $\forall x \in X, \left(\mu_{\widetilde{A}^i}(x)\right) \le \left(\mu_{\widetilde{B}^i}(x)\right)$ and $\left(\nu_{\widetilde{A}^i}(x)\right) \ge \left(\nu_{\widetilde{B}^i}(x)\right)$

Definition 2.3 [4] An intuitionistic fuzzy set is said to be an intuitionistic fuzzy number if it has the following properties:

1. It is an intuitionistic fuzzy subset of the real line.
2. It is normal that is, there is some $x_0 \in R$ such that $\mu_{\widetilde{A}^i}(x_0) = 1$ and $\nu_{\widetilde{A}^i}(x_0) = 0$.
3. It is convex for the membership function $\mu_{\widetilde{A}^i}(x)$ that is

$$\mu_{\widetilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \ge \min\left(\mu_{\widetilde{A}^i}(x_1), \mu_{\widetilde{A}^i}(x_2)\right) \forall x_1, x_2 \in R, \lambda \in [0, 1]$$

4. It is concave for non-membership function $\nu_{\widetilde{A}^i}(x)$ that is

$$\nu_{\widetilde{A}^i}(\lambda x_1 + (1 - \lambda x_2)) \le \max\left(\nu_{\widetilde{A}^i}(x_1), \nu_{\widetilde{A}^i}(x_2)\right) \forall x_1, x_2 \in R, \lambda \in [0, 1]$$

Definition 2.4 [5] Triangular intuitionistic fuzzy number (TIFN) $\widetilde{A}^i$ is denoted by

$$\widetilde{A}^i = \left\langle (a_1, a_2, a_3)\left(a_1', a_2, a_3'\right) \right\rangle$$

such that $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$, where the membership function

$$\mu_{\widetilde{A}^i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

and non-membership function

$$\nu_{\widetilde{A}^i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & \text{for } a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2} & \text{for } a_2 \leq x \leq a_3' \\ 1 & \text{otherwise} \end{cases} \tag{2.4}$$

3 Components of the Proposed Intuitionistic Fuzzy Inference System (IFIS)

The basic components of the proposed intuitionistic fuzzy inference system are shown in Fig. 1.

In this intuitionistic fuzzy inference system (IFIS), we consider two input variables as current income (CI), total net worth (TNW) and one output variable as risk tolerance (RT) level. In this problem, we consider five linguistic variables as VL: very low, L: low, M: medium, H: high, and VH: very high. All linguistic variables are taken as triangular intuitionistic fuzzy numbers.

Current income (CI) = {VL, L, M, H, VH}
Total net worth (TNW) = {VL, L, M, H, VH}
Risk tolerance (RT) = {VL, L, M, H, VH}

The range of input and output variables taken as [3]

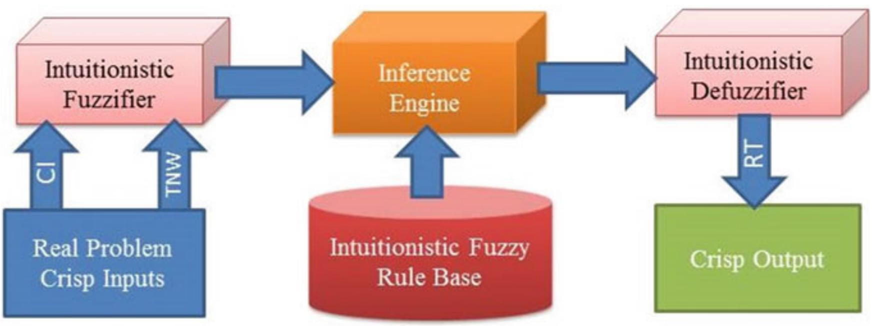


Fig. 1 Intuitionistic fuzzy inference system

$$\begin{aligned} &\text{CI} = \{x * 10^3 : 0 \leqslant x \leqslant 100\}, \text{TNW} = \{y * 10^5 : 0 \leqslant y \leqslant 100\}, \\ &\text{RT} = \{z : 0 \leqslant z \leqslant 100\}, \end{aligned},$$

where x, y are real numbers in \$ and z represent the risk % between 0 and 100.

4 Fuzzification of Input and Output Variables

The membership and non-membership functions of input variable current income (CI) are represented as

$\mu_{\text{VL}}(x) = \begin{cases} \frac{12-x}{12-0} & \text{for } 0 \le x \le 12 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{VL}}(x) = \begin{cases} \frac{x-0}{15-0} & \text{for } 0 \le x \le 15 \\ 1 & \text{otherwise,} \end{cases}$
$\mu_{\text{L}}(x) = \begin{cases} \frac{x-10}{20-10} & \text{for } 10 \le x \le 20 \\ \frac{40-x}{40-20} & \text{for } 20 \le x \le 40 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{L}}(x) = \begin{cases} \frac{20-x}{20-8} & \text{for } 8 \le x \le 20 \\ \frac{x-20}{42-20} & \text{for } 20 \le x \le 42 \\ 1 & \text{otherwise,} \end{cases}$
$\mu_{\text{M}}(x) = \begin{cases} \frac{x-30}{50-30} & \text{for } 30 \le x \le 50 \\ \frac{70-x}{70-50} & \text{for } 50 \le x \le 70 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{M}}(x) = \begin{cases} \frac{50-x}{50-28} & \text{for } 28 \le x \le 50 \\ \frac{x-50}{73-50} & \text{for } 50 \le x \le 73 \\ 1 & \text{otherwise,} \end{cases}$
$\mu_{\text{H}}(x) = \begin{cases} \frac{x-60}{75-60} & \text{for } 60 \le x \le 75 \\ \frac{85-x}{85-75} & \text{for } 75 \le x \le 85 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{H}}(x) = \begin{cases} \frac{75-x}{75-58} & \text{for } 58 \le x \le 75 \\ \frac{x-75}{87-75} & \text{for } 75 \le x \le 87 \\ 1 & \text{otherwise,} \end{cases}$
$\mu_{\text{VH}}(x) = \begin{cases} \frac{x-80}{100-80} & \text{for } 80 \le x \le 100 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{VH}}(x) = \begin{cases} \frac{100-x}{100-78} & \text{for } 78 \le x \le 100 \\ 1 & \text{otherwise.} \end{cases}$

The membership and non-membership functions of input variable current income (CI) represented using MATLAB are such as Figs. 2 and 3.

The membership and non-membership functions of the another input variable total net worth (TNW) are represented as

$\mu_{\text{VL}}(x) = \begin{cases} \frac{15-x}{15-0} & \text{for } 0 \le x \le 15 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{VL}}(x) = \begin{cases} \frac{x-0}{17-0} & \text{for } 0 \le x \le 17 \\ 1 & \text{otherwise,} \end{cases}$
$\mu_{\text{L}}(x) = \begin{cases} \frac{x-8}{25-8} & \text{for } 8 \le x \le 25 \\ \frac{35-x}{35-25} & \text{for } 25 \le x \le 35 \\ 0 & \text{otherwise,} \end{cases}$	$\nu_{\text{L}}(x) = \begin{cases} \frac{25-x}{25-8} & \text{for } 8 \le x \le 25 \\ \frac{x-25}{36-25} & \text{for } 25 \le x \le 36 \\ 1 & \text{otherwise,} \end{cases}$

(continued)

(continued)

$\mu_{\mathrm{M}}(x)=\begin{cases}\frac{x-30}{45-30} & \text{for } 30\le x\le 45\\ \frac{65-x}{65-45} & \text{for } 45\le x\le 65\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{M}}(x)=\begin{cases}\frac{45-x}{45-28} & \text{for } 28\le x\le 45\\ \frac{x-45}{67-45} & \text{for } 45\le x\le 67\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{H}}(x)=\begin{cases}\frac{x-50}{70-50} & \text{for } 50\le x\le 70\\ \frac{80-x}{80-70} & \text{for } 70\le x\le 80\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{H}}(x)=\begin{cases}\frac{70-x}{70-48} & \text{for } 48\le x\le 70\\ \frac{x-70}{82-70} & \text{for } 70\le x\le 82\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{VH}}(x)=\begin{cases}\frac{x-75}{100-75} & \text{for } 75\le x\le 100\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{VH}}(x)=\begin{cases}\frac{100-x}{100-73} & \text{for } 73\le x\le 100\\ 1 & \text{otherwise.}\end{cases}$

The membership and non-membership functions of the output variable risk tolerance (RT) are represented as

$\mu_{\mathrm{VL}}(x)=\begin{cases}\frac{20-x}{20-0} & \text{for } 0\le x\le 20\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{VL}}(x)=\begin{cases}\frac{x-0}{22-0} & \text{for } 0\le x\le 22\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{L}}(x)=\begin{cases}\frac{x-15}{30-15} & \text{for } 15\le x\le 30\\ \frac{40-x}{40-30} & \text{for } 30\le x\le 40\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{L}}(x)=\begin{cases}\frac{30-x}{30-13} & \text{for } 13\le x\le 30\\ \frac{x-30}{42-30} & \text{for } 30\le x\le 42\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{M}}(x)=\begin{cases}\frac{x-35}{45-35} & \text{for } 35\le x\le 45\\ \frac{60-x}{60-45} & \text{for } 45\le x\le 60\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{M}}(x)=\begin{cases}\frac{45-x}{45-32} & \text{for } 32\le x\le 45\\ \frac{x-45}{63-45} & \text{for } 45\le x\le 63\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{H}}(x)=\begin{cases}\frac{x-50}{70-50} & \text{for } 50\le x\le 70\\ \frac{80-x}{80-70} & \text{for } 70\le x\le 80\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{H}}(x)=\begin{cases}\frac{70-x}{70-48} & \text{for } 48\le x\le 70\\ \frac{x-70}{85-70} & \text{for } 70\le x\le 85\\ 1 & \text{otherwise,}\end{cases}$
$\mu_{\mathrm{VH}}(x)=\begin{cases}\frac{x-75}{100-75} & \text{for } 75\le x\le 100\\ 0 & \text{otherwise,}\end{cases}$	$\nu_{\mathrm{VH}}(x)=\begin{cases}\frac{100-x}{100-72} & \text{for } 72\le x\le 100\\ 1 & \text{otherwise.}\end{cases}$

5 Intuitionistic Fuzzy Inference Rules

In this model, we consider two inputs heaving five linguistic variables each. There are total twenty five, if and then rules that are used for intuitionistic fuzzy inference shown in the table below

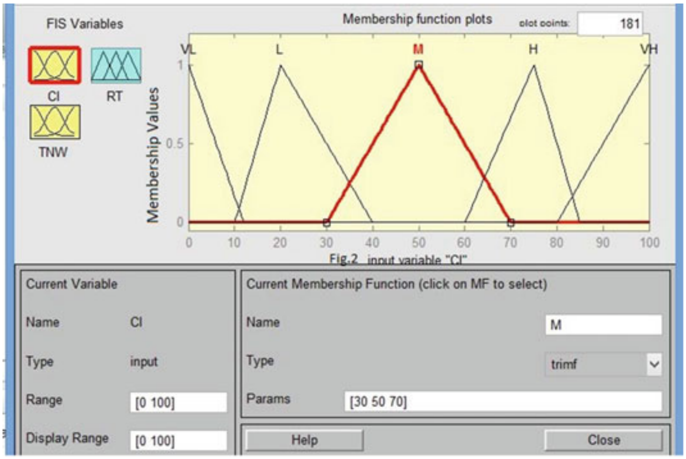


Fig. 2 Input variable “CI”

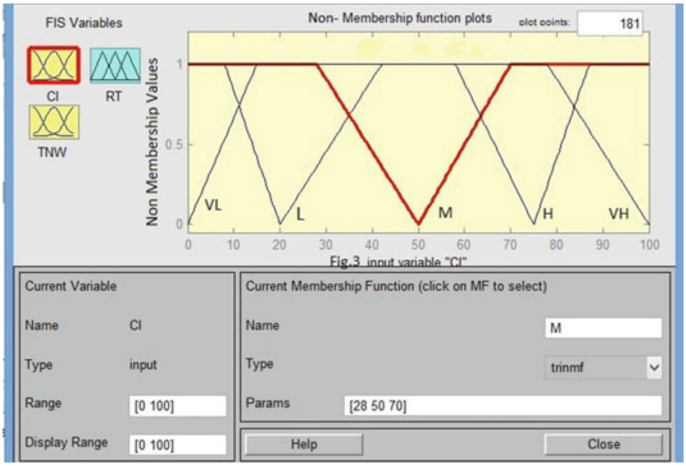


Fig. 3 Input variable “CI”

CI	TNW				
	VL	L	M	H	VH
VL	VL	VL	L	L	M
L	VL	L	L	M	M
M	L	L	M	M	H
H	L	M	M	H	VH
VH	M	M	H	VH	VH

Rule (1): If (CI is VL) and (TNW is VL), then (RT is VL)
Rule (2): If (CI is VL) and (TNW is L), then (RT is VL)
...
Rule (25): (CI is VH) and (TNW is VH), then (RT is VH).

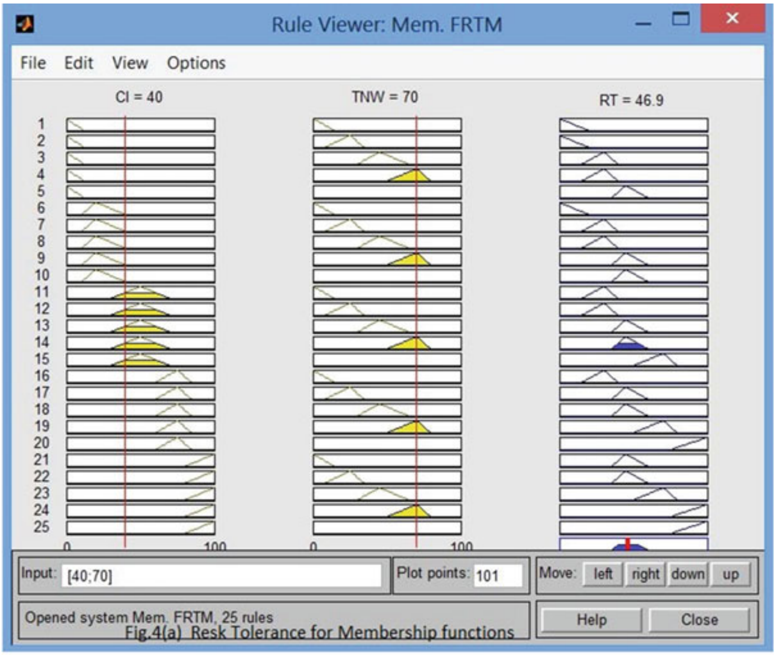
6 Defuzzification Using Centroid Method (COA)

To execute our intuitionistic fuzzy inference model, we are choosing the value of input variables randomly. Let current income (CI) = 40 and total net worth (TNW) = 70 (Fig. 4).

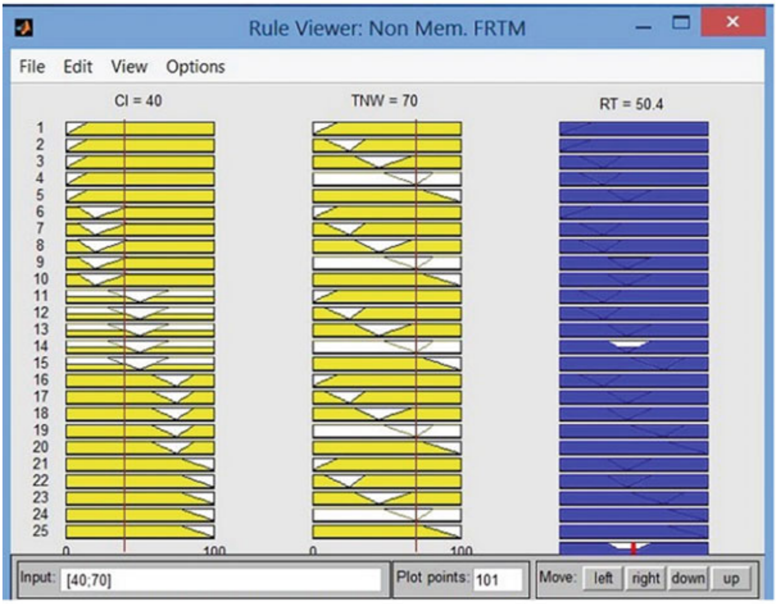
Using if and then rules we get the result risk tolerance (RT) is 46.9 for membership function and 50.4 for non-membership function (Fig. 5).

7 Conclusion

An intuitionistic fuzzy control system provides a flexible model to elaborate the uncertainty and vagueness involved in real-world problems. In this paper, we proposed to develop the non-membership function and defuzzification using MATLAB and applied it in a business model to calculate the risk tolerance based on current income and total net worth. In this paper, we choose a random value of current income (CI) as 40 and total net worth (TNW) as70 of a businessman. We find that acceptance of risk tolerance (RT) is 46.9% and non-acceptance of risk tolerance is 50.4%.The remaining 2.7% is doubtful it may be not maybe acceptance of risk tolerance. We observed that the non-membership functions may improve the performance of an intuitionistic fuzzy control system.



(a)



(b)

Fig. 4 **a** Risk tolerance for membership function. **b** Risk tolerance for non-membership function

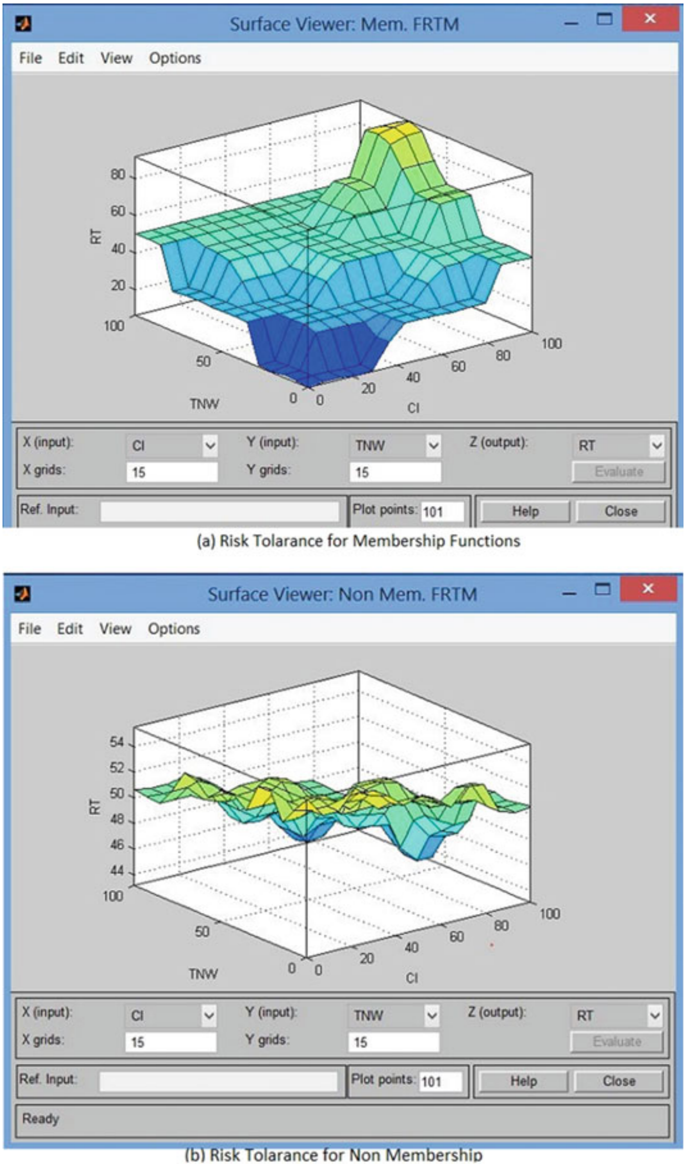


Fig. 5 **a** Risk tolerance for membership function. **b** Risk tolerance for non-membership function

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