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COMPUTER TECHNOLOGIES IN PHYSICS

Approximation of the Dependence of the Radius of the Atomic Nucleus on Its Parameters Using a Fuzzy Hybrid Network Model

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Abstract—In the present investigation, the model of the hybrid network in the form of an adaptive system neuro-fuzzy inference is developed for approximating the dependence of the radius of an atomic nucleus vs. its charge and mass number. With the use of developed model, the radii of 84 nuclides for which experimental data on their sizes were not available were estimated.

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INTRODUCTION

Presently, data on the sizes of atomic nuclei obtained using various experimental methods is known. Due to the absence of the single theory of nuclear forces, several empirical formulas were suggested for determining the nucleus radius by its parameters [1–3]. The standard deviation of radius values obtained using these formulas lies within the interval from 0.106 to 0.023 fm.

Experimental data on radii of nuclei constantly become more and more exact with the help of the combined consideration of nuclei sizes obtained by various methods [4, 5].

Artificial neural networks are widely used in economics, technology, medicine, geology, and physics [6, 7]. Data is successfully being approximated using fuzzy neural networks [8, 9].

The most exact results on measuring nuclei sizes were obtained during the scattering of fast electrons on nuclei and from mesoatom spectra. There measurements provide so called root-mean-square electric radius of the nucleus. Our goal was to create a fuzzy model of a hybrid network which could be used to approximate the dependency of the root-mean-square electric radius of a nucleus on its charge and mass numbers.

1. DEVELOPMENT OF A FUZZY MODEL OF A HYBRID NETWORK

Hybrid network represents a multilayer neural network of special structure without feedbacks. The fuzzy model of a hybrid network uses an existing data sample for defining parameters of a membership function which correspond best of all to some fuzzy logic system. At the same time, the parameters of the membership function are defined using neural-network training procedures.

We used the Fuzzy Logic Toolbox package of the MATLAB environment to develop a fuzzy model of a hybrid network. Hybrid networks in this package are achieved in the form of adaptive neural fuzzy logic ANFIS. Hybrid network ANFIS is a symbiosis of neural network with a Sugeno type fuzzy logic system of first or second order. The neural network has a single output and several inputs, which are fuzzy linguistic variables. Each rule of fuzzy productions in a fuzzy logic system has a weight equal to 1.

We selected two variables as input data: charge number Z and mass number A. The output variable was represented by the root-mean-square radius of nucleus R. In work [10] we used experimental data for 810 nuclides (from ${}^{1}_{1}\mathrm{H}$ to ${}^{248}_{96}\mathrm{Cm}$). Training and test samples included 752 and 58 nuclides, correspondingly.

The fuzzy logic system in our case is based on a simple method of lattice partition, according to which membership functions of fuzzy terms are uniformly distributed within the range of data change. The knowledge base contains all possible variants of rules. Coefficients in rule conclusions are accepted as equal to zero.

The membership function for the output variable was represented by a linear function. For input variables we composed combinations, where the number of linguistic terms varied from 3 to 7 and membership functions were triangular, trapezoidal, generalized bell-curve, and Gaussian, as well as two-side Gaussian, which were set in sequence.

A fuzzy hybrid network was set up using the hybrid training method, which is a combination of the least

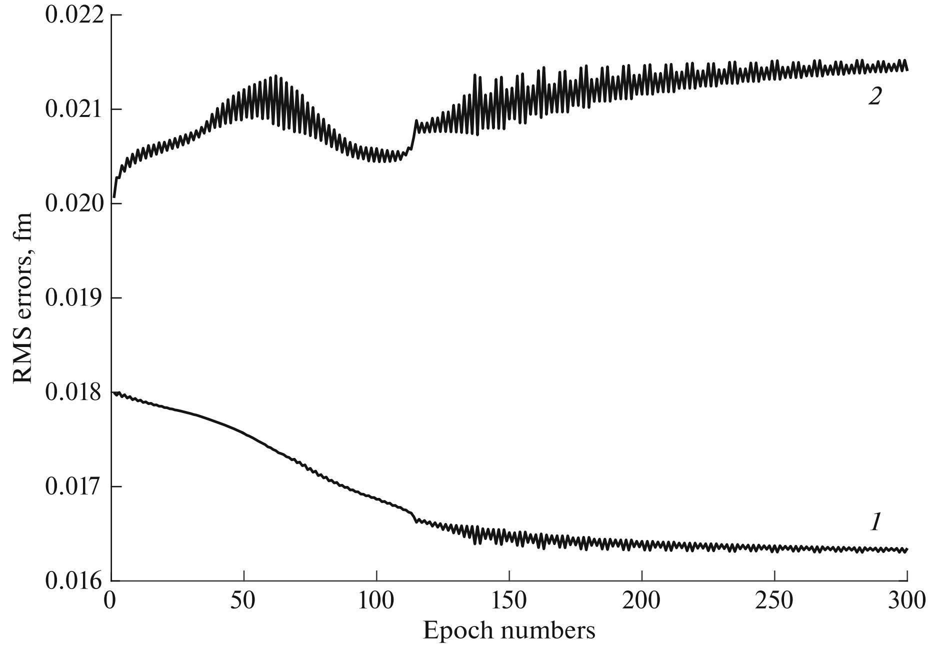

Fig. 1. Error dynamics of fuzzy simulation based on number of training iterations: (*1*) error on training sample; (*2*) error on test sample.

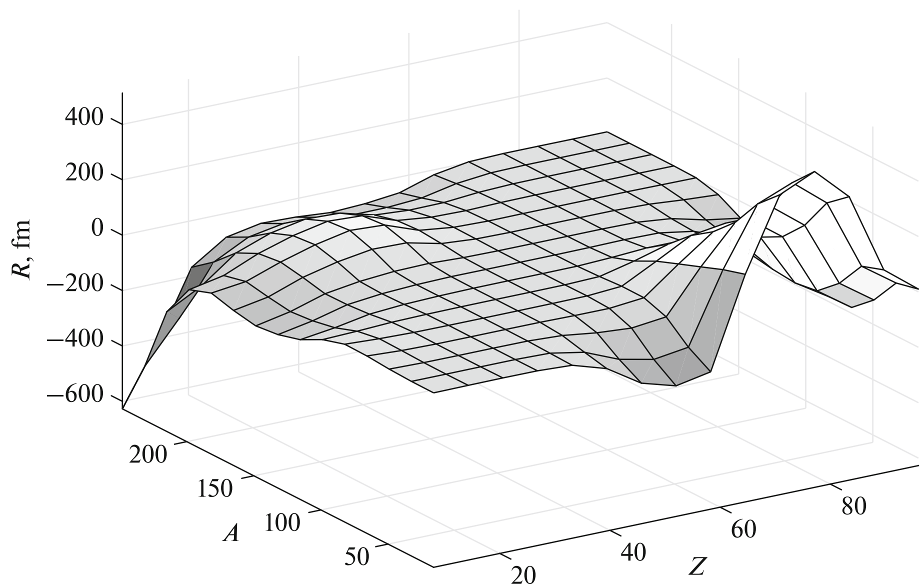

Fig. 2. Surface inputs–output of the fuzzy model after 110 training iterations.

square method and inverse gradient decreasing method, with an error level of 0. Training was carried out over 200 iterations. After that, the models were verified on a test sample.

The best result was obtained for a number of linguistic terms of 5 for each input variable and using the generalized bell-curve function of the membership. At the same time, the root-mean-square error on training and test samples constituted 0.039 and 0.041 fm, correspondingly.

A model of a fuzzy hybrid network with such values of root-mean-square error cannot be used for defining radius of nuclides, which do not have experimental values. An analysis of the developed model made it possible to conclude that quite large errors on samples are conditioned by peculiarities of experimental data. Indeed, properties of light atoms differ from general patterns typical for average and heavy nuclei; it is desirable to group nuclides by mass numbers before developing a fuzzy logic model.

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