# International Scientific Conference

## Algebraic and Geometric Methods of Analysis

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## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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# Holomorphically Projective Mappings of Kähler Manifolds Preserving The Generalized Einstein Tensor

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Holomorphically projective mappings which preserved the Einstein tensor

$$E_{ij} = R_{ij} - \frac{Rg_{ij}}{n}$$

were studied in [1]. Preserving the stress-energy tensor

$$S_{ij} = R_{ij} - \frac{Rg_{ij}}{2}$$

by conformal mappings was explored in [3], [5]. It's worth for noting that in many classical issues e. g. [2, p. 359], just the latter is referred to as the Einstein tensor.

Let us refer to

$$\mathfrak{E}_{ij} \overset{\text{def}}{=} R_{ij} - \kappa R g_{ij}. \tag{1}$$

as **the generalized Einstein tensor**. Here $\kappa$ is a constant. Conformal mappings which preserving the introduced tensor were explored in [6].

It is known that a covariant vector $\psi_i$ determining holomorphically projective mapping between two Kähler spaces $(V^n, J, g)$ and $(\overline{V}^n, J, \overline{g})$ should satisfy the equations

$$\psi_{i,j} = \psi_i\psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{1}{n+2}(\overline{R}_{ij} - R_{ij}). \tag{2}$$

Here we denote by comma "," covariant derivative respect to the metric $g$ of a space $(V^n, J, g)$. The affinor $J_i^h$ is referred to as a *complex structure*. The structure is the same for both manifolds. The symbols $R_{ij}$ and $\overline{R}_{ij}$ denote Ricci tensors of spaces $(V^n, J, g)$ and $(\overline{V}^n, J, \overline{g})$ respectively.

It follows from (38) that the deformation of the generalized Einstein tensor can be written as

$$\overline{\mathfrak{E}}_{ij} - \mathfrak{E}_{ij} = \overline{R}_{ij} - \kappa\overline{R}\overline{g}_{ij} - R_{ij} + \kappa R g_{ij}. \tag{3}$$

Taking account of the preservation requirement, i. e. $\overline{\mathfrak{E}}_{ij} = \mathfrak{E}_{ij}$, from (38) we get

$$\overline{R}_{ij} - R_{ij} = \kappa\overline{R}\overline{g}_{ij} - \kappa R g_{ij}. \tag{4}$$

Since (38) holds we can rewrite (38) as

$$\psi_{i,j} = \psi_i\psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{\kappa}{n+2}(\overline{R}\overline{g}_{ij} - Rg_{ij}). \tag{5}$$

Let us recall that $R = R_{ij}g^{ij}$.

Differentiating (38) covariantly with respect to $x^k$ and the connection $\Gamma$ which is compatible with the metric $g$ of the manifold $(V^n, J, g)$, alternating in $j$ and $k$ and using the Ricci identity, we obtain the conditions of integrability:

$$\psi_\alpha W_{ijk}^\alpha = \frac{\kappa}{n+2}(\partial_k \overline{R}\overline{g}_{ij} - \partial_j \overline{R}\overline{g}_{ik} - \partial_k R g_{ij} + \partial_j R g_{ik}), \tag{6}$$

where

$$W_{ijk}^h \stackrel{\text{def}}{=} R_{ijk}^h + \frac{\kappa R}{n+2}(\delta_j^h g_{ik} - \delta_k^h g_{ij} - J_j^h J_{ik} + J_k^h J_{ij} - 2J_i^h J_{jk}). \tag{7}$$

Finally, we can summarize by the theorem

**Theorem 1.** *If manifolds $(V^n, J, g)$ and $(\overline{V}^n, J, \overline{g})$ are in holomorphically projective correspondence and the mapping preserves the tensor $\mathfrak{E}_{ij} = R_{ij} - \kappa R g_{ij}$, then the function $\psi$ generating the mapping, must satisfy the system of PDE's* (38) *whose conditions of integrability are* (38)*. Also, the tensor $W_{ijk}^h$ is preserved by the mapping.*

## References


[1] Chepurna O., Mikeš J. Holomorphically projective mappings preserving the Einstein tensor. *J. Appl. Math. Aplimat* 2011, 4:2, 187-192.

[2] Petrov A. Z. New methods in the general theory of relativity, *Nauka* 1966, 496 p.

[3] Conformal mappings preserving the stress-energy tensor *Izvestia PSPU* 2011, vol. 26, p. 98-104.

[4] Chepurna O., Kiosak V., Mikeš J. Conformal mappings of Riemannian spaces which preserve the Einstein tensor *J. Appl. Math. Aplimat* 2010, 3:1, p. 253-258.

[5] Cherevko Y. V. Conformal transformations preserving the stress-energy tensor *Proc. Inter. Geom. Center* 2012, vol. 5, no. 3-4, p. 51-58.

[6] Cherevko Y., Chepurna O., Berezovski V., Conformal mappings of Riemannian manifolds preserving the generalized Einstein tensor *APLIMAT 2018 - Proceedings* 2018, p. 224-231.