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On Canonical F-planar Mappings of Spaces with Affine Connection

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Abstract. In this paper we study the theory of F-planar mappings of spaces with affine connection. We obtained condition, which preserved the curvature tensor. We also studied canonical F-planar mappings of space with affine connection onto symmetric spaces. In this case, the main equations have the partial differential Cauchy type form in covariant derivatives. We got the set of substantial real parameters on which depends the general solution of that PDE's system.

1. Introduction

In this paper, we studied F-planar mappings of spaces with affine connection. This theory is a natural continuation of work by Levi-Civita, [15]. The theory of geodesic mappings has been developed by many people, for example Y. Thomas, H. Weyl, P.A. Shirokov, A.S. Solodnikov, A.Z. Petrov, N.S. Sinyukov, A.V. Aminova, J. Mikeš, S. Formella see [1, 10, 17, 19, 21, 22, 28, 33]. There were many questions in the theory of geodesic mappings, which were developed by V.F. Kagan, G. Vranceanu, Y.L. Shapiro, D.V. Vedenyapin etc. These authors found the special classes of $(n-2)$- projective spaces.

The *quasi geodesic mappings* was defined by A.Z. Petrov and they are very close to holomorphically projective mappings of Kähler spaces studied by T. Otsuki, Y. Tashiro, M. Prvanović, J. Mikeš etc., see [18, 19, 22, 26, 30, 33, 38].

The natural generalization of above mentioned mappings are almost geodesic mappings defined by N.S. Sinyukov, see [19, 33]. He distinguished three kinds of almost geodesic mappings, namely π_1, π_2 and π_3 [33, 34]. One should note that these types can intersect. V.E. Berezovski and J. Mikeš [3, 5, 6, 19] proved, that only three types of those mappings can exist. Next, who developed these mappings were V.S. Sobčuk, N.Y. Yablonskaya, V.E. Berezovski, J. Mikeš, M.S. Stanković, L.M. Velimirović, M.L. Zlatanović, N.O. Vesić, V.M. Stanković, etc. [4, 18, 29, 35–37, 42, 44–47].

F-planar mappings were defined by J. Mikeš and N.S. Sinyukov [22, 25] as the widest possible generalization of geodesic, quasi-geodesic and almost geodesic mappings of π_2 type.

The basic equations of F-planar mappings, which were obtained in the work by J. Mikeš and N.S. Sinyukov [25], have recently been clarified in [12]. The next study of F-planar mappings is possible to find

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in [14] and also in [2, 19]. In the paper [12], there was proved that PQ^{ε}-equivalence, defined by P. Topalov, are previously studied F_2-planar mappings. For this reason, we introduced the concept of F_{ε}^2-planar mappings.

In this paper, we study the theory of F-planar mappings of spaces with affine connection. We obtained condition, which preserved the curvature tensor. We also study conditions, when a space A_n with affine connection admits the canonical F-planar mapping. Those conditions has closed form of partial differential Cauchy-like system in covariant derivatives. On the base of that system, we set the number of real parameters on which the general solution depends. In whole work, we use the local tensor notation and also suppose that the class of a functions is smooth enough.

2. Basic concepts of the theory of F-planar mappings of space with affine connection

Now, we will provide the basic definitions and properties of F-planar mappings, which is possible to find in monograph [19, p. 385], and in the paper [12].

Let $A_n = (M, \nabla, F)$ be an n-dimensional manifold M with affine connection ∇, and affinor structure F, i.e. a tensor field of type $(1, 1)$.

Definition 2.1 (Mikeš, Sinyukov [25], see [22, p. 213]). A curve ℓ, which is given by the equations $\ell = \ell(t)$, $\lambda(t) = d\ell(t)/dt$ $(\neq 0)$, $t \in I$, where t is a parameter, is called an *F-planar*, if its tangent vector $\lambda(t_0)$, for any initial value t_0 of the parameter t, remains, under parallel translation along the curve ℓ, in the distribution generated by the vector functions λ and $F\lambda$ along ℓ.

In accordance with this definition, ℓ is F-planar if and only if the following condition holds:

$$\nabla_{\lambda(t)}\lambda(t) = \varrho_1(t)\lambda(t) + \varrho_2(t)F\lambda(t),$$

where ϱ_1 and ϱ_2 are some functions of the parameter t, see ([25], [22, p. 213]).

We suppose two spaces A_n and $\bar{A}_n$ with torsion-free affine connection ∇ and $\bar{\nabla}$, respectively. Affine structures F and $\bar{F}$ are defined on A_n, resp. $\bar{A}_n$.

Definition 2.2 (Mikeš, Sinyukov [25], see [22, p. 213]). A diffeomorphism f between manifolds with affine connection A_n and $\bar{A}_n$ is called an *F-planar mapping* if any F-planar curve in A_n is mapped onto an $\bar{F}$-planar curve in $\bar{A}_n$.

Due to the diffeomorphism f, we always suppose that ∇, $\bar{\nabla}$, and the affinors F, $\bar{F}$ are defined on M $(\bar{M})$ where $A_n = (M, \nabla, F)$ and $\bar{A}_n = (M, \bar{\nabla}, \bar{F})$.

The diffeomorphism $f\colon A_n \to \bar{A}_n$ is an F-planar mapping if and only if for the deformation tensor $P = \bar{\nabla} - \nabla$ of mapping f, the following conditions holds

$$P(X, Y) = \psi(X) \cdot Y + \psi(Y) \cdot X + \varphi(X) \cdot F(Y) + \varphi(Y) \cdot F(X), \tag{1}$$

for any tangent vectors X, Y, where ψ, φ are linear forms. Moreover, this mapping preserves affinor structures $\bar{F} = \alpha \cdot F + \beta \cdot I$, see [12, 25], [19, p. 385-392].

In the common coordinate system $x = (x^1, x^2, \ldots, x^n)$, respective F-planar mapping, formula (1) can be rewritten:

$$P_{ij}^h(x) = \delta_i^h\psi_j + \delta_j^h\psi_i + F_i^h\varphi_j + F_j^h\varphi_i, \tag{2}$$

where $\psi_i(x)$, $\varphi_i(x)$ are components of linear forms ψ and φ.

F-planar mapping is called *canonical*, if the form ψ in equation (2) vanishes. Evidently, F-planar mapping can be expressed as a composition of geodesic and canonical F-planar mappings.

In the common coordinate system $x = (x^1, x^2, \ldots, x^n)$, the canonical F-planar mapping $f\colon A_n \to \bar{A}_n$ is characterized by the following conditions

$$P_{ij}^h = F_{(i}^h\varphi_{j)}. \tag{3}$$

3. Canonical *F*-planar mappings with *e*-structures

We suppose that the affinor structure F defined in the space A_n fulfilles condition $F^2 = e \cdot Id$, where $e = \pm 1$. In coordinate form

$$F^h_\alpha F^\alpha_i = e\,\delta^h_i. \tag{4}$$

Those structures are called *e-structures* [33]. In this case, we will sign F-planar mapping as $\pi(e)$, $e = \pm 1$.

Next, we will study canonical F-planar mapping $\pi(e)$ which is characterized by conditions (3) and (4). In the work [7], we have proved that in F-planar mapping the curvature tensor is preserved if and only if it satisfies

$$A^h_{ijk} = A^h_{ikj}, \tag{5}$$

where

$$A^h_{ijk} \equiv P^h_{ij,k} + P^\alpha_{ij}\,P^h_{\alpha k}. \tag{6}$$

From formula (3) for $\pi(e)$, $e = \pm 1$, we get

$$A^h_{ijk} = \varphi_{i,k}F^h_j + \varphi_i F^h_{j,k} + \varphi_{j,k}F^h_i + \varphi_j F^h_{i,k} + \varphi_i\varphi_\alpha F^\alpha_j F^h_k + \varphi_j\varphi_\alpha F^\alpha_i F^h_k + \varphi_j\varphi_k F^\alpha_i F^h_\alpha. \tag{7}$$

Using formula (4), the above formula will be simplified:

$$A^h_{ijk} = \varphi_{i,k}F^h_j + \varphi_{j,k}F^h_i + \varphi_i(F^h_{j,k} + \varphi_\alpha F^\alpha_j F^h_k + e\delta^h_j\varphi_k) + \varphi_j(F^h_{i,k} + \varphi_\alpha F^\alpha_i F^h_k + e\delta^h_i\varphi_k). \tag{8}$$

Substituting (8) to (5), we get

$$\varphi_{i,k}F^h_j - \varphi_{i,j}F^h_k + \varphi_{j,k}F^h_i - \varphi_{k,j}F^h_i = B^h_{ijk}, \tag{9}$$

where

$$B^h_{ijk} = \varphi_k(F^h_{i,j} + \varphi_\alpha F^\alpha_i F^h_j + e\delta^h_i\varphi_j) + \varphi_i(F^h_{k,j} + \varphi_\alpha F^\alpha_k F^h_j + e\delta^h_k\varphi_j - F^h_{j,k} - \varphi_\alpha F^\alpha_j F^h_k - e\delta^h_j\varphi_k) - \varphi_j(F^h_{i,k} + \varphi_\alpha F^\alpha_i F^h_k + e\delta^h_i\varphi_k). \tag{10}$$

We contract equation (9) with the structure F^h_ρ, respective indices h and ρ. Finaly, we have

$$e\varphi_{i,k}\delta^m_j - e\varphi_{i,j}\delta^m_k + e\varphi_{j,k}\delta^m_i - e\varphi_{k,j}\delta^m_i = B^\alpha_{ijk}F^m_\alpha,$$

or equivalently

$$\varphi_{i,k}\delta^m_j - \varphi_{i,j}\delta^m_k + \varphi_{j,k}\delta^m_i - \varphi_{k,j}\delta^m_i = eB^\alpha_{ijk}F^m_\alpha. \tag{11}$$

Now, we contract the last formula with respect to indices m and j. We get

$$n\,\varphi_{i,k} - \varphi_{k,i} = eB^\alpha_{i\beta k}F^\beta_\alpha. \tag{12}$$

After alternation with respect to the indices i and k, it takes

$$\varphi_{i,k} - \varphi_{k,i} = \frac{e}{n+1}\,(B^\alpha_{i\beta k} - B^\alpha_{k\beta i}). \tag{13}$$

On the base of formula (13), the condition (12) can be expressed:

$$\varphi_{i,k} = \frac{e}{n-1}\,F^\beta_\alpha\,(B^\alpha_{i\beta k} - \frac{1}{n+1}\,B^\alpha_{k\beta i} - B^\alpha_{i\beta k}). \tag{14}$$

Let us suppose that the structure F and its covariant derivative in A_n are apriori given. From above, it follows

Theorem 3.1. *Let the $\pi(e)$, $e = \pm 1$ be a canonical F-planar mapping A_n onto $\bar{A}_n$ preserving the curvature tensor. Then the formula* (14) *is necessary and sufficient condition for partial differential equation of Cauchy-like system, respective functions $\varphi_i(x)$.*

A general solution of that system depends on no more than n real parameters.

4. Canonical F-planar mappings $\pi(e)$, $e = \pm 1$, of space with affine connection onto symmetric spaces

A space with affine connection is called (*locally*) *symmetric* if the curvature tensor is absolutely parallel, see P.A. Shirokov [32], É. Cartan [8], S. Helgason [11]. Those spaces have a great importance in the theory of geodesic, holomorphically projective mappings of symmetric spaces, see [9, 13, 17, 18, 23, 31, 33, 34].

We will study the canonical F-planar mappings $\pi(e)$, $e = \pm 1$, of spaces with affine connection A_n onto symmetric spaces $\bar{A}_n$ which are characterized by the condition

$$\bar{R}^h_{ijk|m} \equiv 0, \tag{15}$$

where $\bar{R}^h_{ijk}$ are components of the curvature tensor on $\bar{A}_n$, and " , " denotes covariant derivative on $\bar{A}_n$.

Let us suppose that A_n and $\bar{A}_n$ have a common coordinate system $x = (x^1, x^2, \dots, x^n)$ with respect to the mapping $\pi(e)$, the structure F is defined on A_n and (4) holds. Because

$$\bar{R}^h_{ijk|m} = \frac{\partial \bar{R}^h_{ijk}}{\partial x^m} + \bar{\Gamma}^h_{m\alpha}\bar{R}^\alpha_{ijk} - \bar{\Gamma}^h_{mi}\bar{R}^h_{\alpha jk} - \bar{\Gamma}^\alpha_{mj}\bar{R}^h_{i\alpha k} - \bar{\Gamma}^\alpha_{mk}\bar{R}^h_{ij\alpha},$$

holds then after substitution by the formula (1), which characterizes the mapping $\pi(e)$, we have

$$\bar{R}^h_{ijk|m} = \bar{R}^h_{ijk,m} + P^h_{m\alpha}\bar{R}^\alpha_{ijk} - P^\alpha_{mi}\bar{R}^h_{\alpha jk} - P^\alpha_{mj}\bar{R}^h_{i\alpha k} - P^\alpha_{mk}\bar{R}^h_{ij\alpha}. \tag{16}$$

Because $\bar{A}_n$ is symmetric, using (15) and conditions (3), from conditions (16), we obtain the following

$$\bar{R}^h_{ijk,m} = \varphi_{(i}F^\alpha_{m)}\bar{R}^h_{\alpha jk} + \varphi_{(m}F^\alpha_{j)}\bar{R}^h_{i\alpha k} + \varphi_{(m}F^\alpha_{k)}\bar{R}^h_{ij\alpha} - \varphi_{(i}F^h_{\alpha)}\bar{R}^\alpha_{ijk}, \tag{17}$$

where the round brackets mean the symmetrization with respect to the given indices.

It is known [22, p. 213] that in A_n and $\bar{A}_n$ there are the following relation between the curvature tensors

$$\bar{R}^h_{ijk} = R^h_{ijk} + P^h_{ik,j} - P^h_{ij,k} + P^\alpha_{ik}P^h_{j\alpha} - P^\alpha_{jk}P^h_{k\alpha}. \tag{18}$$

After some calculations, from (3), conditions (18) takes form

$$\varphi_{i,j}F^h_k + \varphi_{k,j}F^h_i - \varphi_{i,k}F^h_j - \varphi_{j,k}F^h_i = C^h_{ijk}, \tag{19}$$

where

$$C^h_{ijk} = \bar{R}^h_{ijk} - R^h_{ijk} - \varphi_i(F^h_{k,j} - F^h_{j,k} + e\delta^h_k\varphi_j + \varphi_\alpha F^\alpha_k F^h_j - e\delta^h_j\varphi_k - \varphi_\alpha F^\alpha_j F^h_k) + \varphi_k(F^h_{i,j} + \varphi_\alpha F^\alpha_i F^h_j) - \varphi_j(F^h_{i,k} + \varphi_\alpha F^\alpha_i F^h_k). \tag{20}$$

Contracting formula (19) with the affinor structure F^h_ρ, respective ρ and h, we have

$$\delta^m_k\phi_{i,j} + \delta^m_i\phi_{k,j} - \delta^m_j\phi_{i,k} - \delta^m_i\phi_{j,k} = eC^\alpha_{ijk}F^m_\alpha. \tag{21}$$

Contracting (21) with respect to the indices m and i, we get

$$\varphi_{k,j} - \varphi_{j,k} = \frac{e}{n+1}\,C^\alpha_{\beta jk}F^\beta_\alpha. \tag{22}$$

Analogically, contracting (21) with respect to the indices k and m, we obtain

$$n\varphi_{i,j} - \varphi_{j,i} = eC^\alpha_{\beta jk}F^\beta_\alpha. \tag{23}$$

Using (22), the (23) is simplified to

$$\varphi_{i,j} = \frac{e}{n-1}\,(C^\alpha_{ij\beta} - \frac{1}{n+1}\,C^\alpha_{\beta ji})F^\beta_\alpha. \tag{24}$$

Formulas (17) and (24) in A_n are forming a closed Cauchy-like system of partial differential equation of unknown functions $\bar{R}^h_{ijk}(x)$ and $\varphi_i(x)$. Because the $\bar{R}^h_{ijk}(x)$ are components of the curvature tensor in $\bar{A}_n$, they have to fulfill following identities

$$\bar{R}^h_{ijk} + \bar{R}^h_{ikj} = 0, \text{ and } \bar{R}^h_{ijk} + \bar{R}^h_{jki} + \bar{R}^h_{kij} = 0. \tag{25}$$

Finaly, we obtain the following.

Theorem 4.1. *A space A_n with affine connection admits the canonical F-planar mapping $\pi(e)$, $e = \pm 1$ onto symmetric space $\bar{A}_n$ if and only if in A_n exists a solution of the mixed Cauchy-like system of the equations* (17), (24) *and* (25), *respective the unknown functions $\bar{R}^h_{ijk}(x)$ and $\varphi_i(x)$.*

It is known that above mentioned system has, for initial condition $\bar{R}^h_{ijk}(x_0)$ and $\varphi_i(x_0)$, more than one solution at the point $x_0 \in A_n$. From this and from conditions (25) it follows that the general solution of such system depends on no more than $\frac{1}{3}\, n^2\, (n^2 - 1)$ real parameters.

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